NOTATION

r, x, coordinates; $\bar{r} = r/R_1$, $\bar{x} = x/R_1$, dimensionless coordinates; τ, time; $Fo = a_1\tau/R_1^2$, Fourier number; λ_1, a_1, c_1, ρ_1, λ_2, a_2, c_2, ρ_2, thermal conductivity, thermal diffusivity, specific heat, density of membrane material and cylinder, respectively; δ_1, membrane thickness; $k_a = a_1/a_2$; $\varepsilon = [\lambda_1 c_1 \rho_1/(\lambda_2 c_2 \rho_2)]^{1/2}$; $\gamma_0 = 2\delta_1 R_1 R_2^{-2}\,[1 - (R_1/R_2)]^{-1}$; $q(r, \tau)$, heat flux; $T_1(r, \tau)$, $T_2(x, \tau)$, temperature of membrane and cylinder; $I_0(z)$, $I_1(z)$, $K_0(z)$, $K_1(z)$, modified Bessel functions and MacDonald functions (of zero and first order).

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CONTROL OF CEMENTATION THROUGH A CONTINUOUS CHANGE IN CARBON POTENTIAL AND TEMPERATURE IN A GAS FURNACE

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A numerical experiment is conducted to study the feasibility of automatic control of cementation through a continuous change in carbon potential and the temperature of the external medium.

1. The idea of controlling the saturation process during cementation by changing the parameters of the furnace atmosphere over time was first proposed in [1]. The authors of [1, 2] examined simple control methods in which the carbon potential of the external medium decreases suddenly at a certain moment. They addressed the question of choosing this moment so that the resulting distribution of carbon in the diffusion layer is as close as possible to a profile with a plateau corresponding to a constant concentration.

The problem of controlling the process with a continuous change in carbon potential over time was solved in [3, 4] by the method of mathematical modeling. It was shown through a mathematical experiment in [3] that changing the duration of the transitional stage makes it possible to regulate the effective thickness of the layer and to approximately attain the necessary carbon distribution in less time. The mathematical notion of regularization [5] was used in [4] to solve the control problem by varying carbon potential. This approach made it possible to fully automate calculation of the optimum control function on a computer.

Here, we make the first attempt to mathematically model the production process (PP) when it is controlled simultaneously by a time change in the carbon potential and the temperature of the saturating atmosphere of the furnace. The control functions were chosen from a class of functions that permits simple realization (Fig. 1) and were parameterized accordingly.

Nomograms were developed in [6] for the simplest method of control — by changing the carbon potential over time. The nomographic method of describing functional relations is especially effective when the functions are explicitly given [7]. However, with the large number of control parameters in our case and the implicit character of the relationship be-

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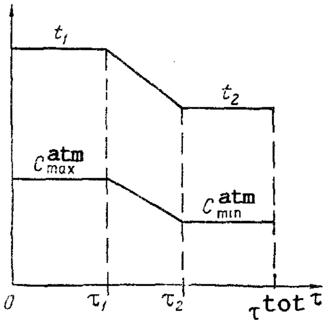

Fig. 1. Model of "smooth" three-stage control of the furnace regime.

tween them and the result of the production process, a complete nomographic description of the process turns out to be impossible. Thus, we used a basically different approach which involved automated computer searching for the necessary regime parameters on the basis of assigned output characteristics of the process. This approach is similar to that used in [4].

The method we used is also promising for controlling saturation in the dynamic regime, i.e. in real time, with the operation of a specialized control computer on the production line.

This does not preclude the use of the nomograms for programmed control of the production process in cases where the program is based on the initial parameters of the process.

A variant of such nomograms, making it possible to assign the duration of the saturation process beforehand within a certain class of simple parameters, is also proposed in the present study.

2. The physical basis of the technology of carbon saturation of a specimen is nonlinear diffusion. We will examine this process in an axisymmetrical approximation, so that the concentration C depends on the space coordinate r and the time τ: $C = C(r, \tau)$, $0 \leqslant r \leqslant R$, $0 \leqslant \tau \leqslant \tau^{tot}$, where R is the radius of the cylindrical specimen and t^{tot} is the total time of saturation of the specimen by carbon.

With each set of control parameters, this function is determined in accordance with the following conditions:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rD(C,\, t)\frac{\partial C}{\partial r}\right) = \frac{\partial C}{\partial \tau} \quad (0 < r < R, \quad 0 < \tau \leqslant \tau^{tot}),$$
$$C|_{\tau=0} = \hat{C}_0, \quad \frac{\partial C}{\partial r}\bigg|_{r=0} = 0, \quad -D\frac{\partial C}{\partial r}\bigg|_{r=R} = \beta(C - C^{atm})|_{r=R}, \tag{1}$$

where C^{atm} is the carbon potential of the furnace atmosphere; β is the mass-transfer coefficient; $\hat{C}_0$ is the initial content of carbon or, for carbon steels, the grade of steel.

Here, the diffusion coefficient depends not only on the concentration — which makes the problem nonlinear — but also on the temperature t of the furnace. The furnace temperature in turn is a function of time: $t = t(\tau)$. Since diffusion processes are considerably slower than thermal processes, we can ignore the time it takes to reach the prescribed temperature. Thus, the process depends on t as a parameter.

Another control parameter is the carbon potential of the furnace atmosphere, which is also dependent on time: $C^{atm} = C^{atm}(\tau)$; this function goes into the boundary condition. The parameter β is independent of concentration.

The following expressions [6] were taken for the material characteristic D and the constant β:

$$D(C,\, t) \equiv D = (0.04 + 0.08C)\exp(-31350/1.987t_k), \ \text{cm}^2/\text{sec}$$
$$\beta = 1.36 \cdot 10^{-3}\exp(-11100/1.987t_k), \ \text{cm/sec}$$
$$t_k = t + 273.15, \ \text{K}.$$

Problem (1) makes it possible to calculate the concentration field for each assigned set of control parameters. We perform these calculations with the use of an implicit finite difference scheme which is of second-order accuracy with respect to the radius and of first-order accuracy with respect to time [8]. The problem is then solvable by the trial run method within the framework of an iterative process. The use of iteration is connected with the nonlinearity of problem (1). Let us turn to the problem of controlling the saturation process.

3. For the control model being examined (Fig. 1) and for the specified grade of steel, we can designate a discrete set of control parameters $\hat{p}$ characterizing the potential and temperature of the furnace atmosphere. Then the concentration field determined by problem (1) at the end of the process $\tau = \tau^{tot}$ turns out to be a functional of $\hat{p}$: $C = C[r, \hat{p}]$.

The desired profile of carbon distribution through the depth of the specimen will be characterized by two reference points. One of these is a point on the surface: $r_0 = R$. This point has an assigned value of concentration ($C^{(0)} = C_{sur}$). The second point is near the surface: $r_1 = R - h_{gr}$. It is associated with the value $C^{(1)} = C_{gr}$. The algorithm that we devised makes it possible to assign any number s of reference points. However, as was shown by numerical experiments we performed, an increase in s does not improve the result. This is not surprising, since we were bound by a constraint when we selected the form of the cementation profile: this is the solution of boundary-value problem (1) with $\tau = \tau^{tot}$. In particular, such a generally-accepted final characteristic as the total cementation depth (h_{tot}, for which $C^{(2)} = \hat{C}_0 + 0.05\%$, [2, 6]) is uncontrolled when a value different from its effective thickness h_{gr} is specified. In this case, it takes the value "ascribed" to it for the chosen parameters $\hat{p}$ of the diffusion law. Thus, the result of the production process is characterized by the preassigned set of values $\hat{q} = \{C_{sur}; h_{gr}, C_{gr}\}$.

Let δ be the allowable standard deviation of the theoretical values of concentration from the required reference-point values. Let there also be assigned a set P from which the control parameters are chosen. Then the quality of the cementation process at any given moment of time can be characterized by the requirement

$$F(\hat{p}) \equiv \sum_{k=0}^{s} \left(\frac{C[r_k, \hat{p}] - C^{(k)}}{C^{(k)}} \right)^2 \leqslant \delta^2, \qquad (2)$$

where $\hat{p} \in P$.

The last condition includes explicit quantitative restrictions on the components $\hat{p}$ which correspond to the values allowable in practice. This ensures correct formulation [5] of quasi-minimization problem (2) in the case of its validity for the given τ^{tot} and δ, i.e. when

$$\inf_P F(\tau^{tot}, \hat{p}) < \delta^2. \qquad (3)$$

On the other hand, the condition $\hat{p} \in P$ may implicitly include a requirement on the qualitative behavior of the cementation profile at the end of the process; monotonicity of the curve C(r), etc. (see Part 4).

The search for $\hat{p} = \hat{p}(\tau^{tot})$ that will ensure the requisite quality is conducted in the automatic regime on a computer in accordance with the prescribed initial approximation $\hat{p}_0$. This search can be done by the formal method in [9], for example. Either the minimization process is interrupted in accordance with condition (2) or we ascertain the independence of the formulation for the given τ^{tot} and δ (condition (3)).

The solution of problem (2) determines the value of the control parameters $\hat{p}$ for which the prescribed quality (δ) is assured by the moment of time τ^{tot}. This may be determined by considerations relating to the organization of the production cycle.

However, we are interested mainly in the problem of choosing the parameters $\hat{p}$ for which a concentration profile of the requisite quality is obtained in the shortest possible time. The objective functional $\tau^{tot}(\hat{p})$ corresponding to this requirement can be determined by the condition $F(\tau^{tot}, \hat{p}) = \delta$, and the optimum (in the sense just indicated) saturation time τ^{tot}_{min} is obtained by solving the following variational problem:

$$\tau^{tot}_{min} = \arg\inf \tau^{tot}(\hat{p}), \quad \hat{p} \in P^{min}_{\delta}, \qquad (4)$$

$$\text{where } P^{min}_{\delta} = \{\hat{p} \in P : F(\tau^{tot}, \hat{p}) = \delta\}.$$

It is natural to seek the solution of this problem on a grid of values $\{\tau_\ell^{tot}\}$. This means that quasi-minimization problem (2) is solved for the remaining control parameters for each τ_ℓ^{tot}. The algorithm for its solution is thus a component part of the complex of methods used to solve the above optimum control problem. This basic part is augmented by the condition for the selection of τ_ℓ^{tot}; we choose the lowest value $\tau_{\ell,min}^{tot} \in \{\tau_\ell^{tot}\}$ for which the condition $\hat{p} \in P_\delta^{min}$ is still satisfied.

Presented below are some results of a mathematical experiment involving an automated search for control parameters which are optimum in the sense of (2) or (4).

4. First we will examine a simple variant of a complex control function in which the temperature in the furnace suddenly changes from t_1 to t_2 ($t_2 < t_1$) at the moment of time τ_2 corresponding to the establishment of C_{min}^{atm}. A reduction in furnace temperature and the corresponding reduction in the temperature of the metal at this stage of the process should reduce diffusion in the surface layers and thus help move carbon deeper into the metal due to the appreciable carbon-content gradient established previously at the boundary of the "effective" layer. A reduction in the temperature of the furnace has the same time of effect as a reduction in the carbon potential of the furnace atmosphere.

With an assigned value of τ^{tot} and a fixed value of t_1: (t_1 = 870°C or 930°C) the control function is characterized by the set $\hat{p} = \{C_{max}^{atm}, C_{min}^{atm}, \tau_1, \tau_2, t_2\}$. The corresponding compact set P of values of $\hat{p}$ will be determined by the production conditions: $1.21\% \geqslant C_{max}^{atm} \geqslant C_{min}^{atm} \geqslant 0.8\%$; $850\,°C \leqslant t_2 < t_1$; $\tau_1 + 15\ \text{min} \leqslant \tau_2 < \tau^{tot}$*).

The solid lines in Fig. 2 show the dependence of cementation profiles which are optimum in the sense (2) on the duration of the carbon saturation process with t_1 = 930°C for $\hat{q} = \{C_{sur} = 0.8\%;\ h_{gr} = 0.8\ \text{mm},\ C_{gr} = 0.6\%\}$ and with $\delta^2 \leq 5\cdot10^{-5}$.

It is possible to discern two characteristic types of carbon distribution in the diffusion layer: the first, corresponding to small values of τ^{tot}, shows a fairly steep linear decrease in concentration with depth; the second, corresponding to large τ^{tot}, is characterized by a surface layer with a concentration which decreases more slowly. Thus, by regulating the cementation time and choosing the appropriate parameters, it is possible to assure a level of quality of carbon distribution in the surface layers which will solve the problem posed initially: provide the given part (specimen) with the requisite strength under intensive impact loading or uniform long-term loading (and, in the latter case, thus improve fatigue strength).

Values of $\hat{p}$ found in the automatic regime are shown in Table 1. Similar behavior is characteristic for other values of $\hat{q}$ in the following ranges of values: C_{sur} = 0.8%; C_{gr} = 0.5, 0.6, 0.7, and 0.8%; h_{gr} = 0.2, 0.5, 0.8, and 1.1 mm.

On the other hand, it turns out that similar cementation results C(r) can be obtained with different sets of control parameters, including different values of the total time τ^{tot}. The corresponding comparisons are made in Fig. 2 (dashed lines) and Table 2. It is this fact that makes it possible to pose the problem of searching for the minimum cementation time without sacrificing quality.

Here, the above-mentioned characteristic features of the profiles make it possible to

TABLE 1. Optimum Values of the Control Parameters with Assigned Reference Points and "Stepped" Control of Temperature in Relation to the Duration of the Process

t_1, °C	τ^{tot}, h	$\hat{q}$, C_{sur} / h_{gr}/C_{gr}	C_{max}^{atm}, % C	C_{min}^{atm}, % C	τ_1, h	τ_2, h	t_2, °C	No. of set
930	10	0,8 / 0,8/0,6	1,14	0,80	7,7	9,3	903,1	1
	20	0,8 / 0,8/0,6	1,13	0,83	8,0	9,3	881,3	2
	30	0,8 / 0,8/0,6	1,14	0,82	1,0	10,0	865,4	3

*In the above calculations, $\hat{C}_0$ = 0.15%, R = 10 mm.

TABLE 2. Two Different Sets of Optimum Parameters Leading to Similar Cementation Profiles with "Stepped" Control

t_1, °C	τ^{tot}, h	$\hat{q}$, $\frac{C_{sur}}{h_{gr}/C_{gr}}$	C^{atm}_{max}, % C	C^{atm}_{min}, % C	τ_1, h	τ_2, h	t_2, °C	No. of set
930	10	$\frac{0,8}{0,5/0,7}$	1,13	0,81	6,9	8,5	850,0	1
870	20	$\frac{0,8}{0,5/0,7}$	1,12	0,81	14,7	16,0	858,5	2

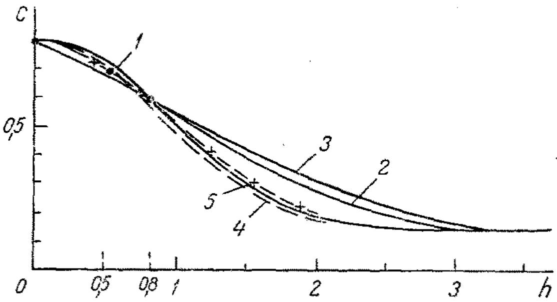

Fig. 2. Cementation profiles with "stepped" control of the temperature regime of the furnace: τ^{tot} = 10 h (1), 20 (2), and 30 h (3); profiles for the first and second sets of parameters from (Table 2) (4) and (5), respectively. C, %; h, mm.

formulate additional conditions determining the set P in order to solve a time optimization problem in the automatic regime. We will use these conditions for a control variant with a continuous change in the temperature of the furnace atmosphere. (In this variant, the temperature t_2 is fixed – t_2 = 840°C – and the control function is characterized by the set $\hat{p} = \{C^{atm}_{max}, C^{atm}_{min}, t_1, \tau_1, \tau_2\}$.)

For profiles that will ensure increased resistance to impact (profile A), the optimality criterion τ^{tot} in an automated search can be the following requirement:

a) the negative derivative of concentration on the surface must be negligible or must differ as little as possible from the slope of the chord passing through the point (C_{sur}, h_{gr} = 0) and (C_{gr}, h_{gr}). Taking into account the allowable standard deviation δ_1 and calculating the derivative on the surface from a certain approximate formula $\tilde{C}_{sur}'$ we find that the imposition of the requirement just stated means that the following inequality is satisfied: $||\tilde{C}'_{sur}| - |\tilde{C}'|| \leq \delta_1$, where $\tilde{C}' = (C_{gr} - C_{sur})/h_{gr}$.

For profiles which solve the problem of increasing fatigue strength (profile B), if we consider the allowable standard deviation δ_2 for the nonuniformity of the carbon distribution in the surface layers, we will judge the distribution and, thus, the corresponding value of τ^{tot} to be optimum when one (or both) of the following criteria is satisfied:

b) $|C_{max} - C_{sur}| \leq \delta_2$,

c) $|\tilde{C}_{sur}| \leq \delta_2$ or the passage of $\tilde{C}'_{sur}(r, \tau^{tot})$ (the derivative on the surface) through zero.

We took the value 0.02% C as δ_1 and δ_2 in subsequent experiments, this value corresponding to the accuracy of measurement of carbon concentration in practice.

Figure 3 shows optimum (with respect to time) concentrations of types A and B for different sets of reference points $\hat{q}$, which are denoted by the dots in the figure. The corresponding approximate values of $\tilde{\tau}^{tot}_{min}$, obtained on the grid: 1 h $\leq \tau^{tot} \leq$ 7 h and 4 h $\leq \tau^{tot}$ $\leq$ 35 h. The values of the control parameters are shown in Table 3.

The results we obtained for different types of control functions show that a smooth change in both control factors – the temperature of the furnace atmosphere and the concentration of carbon in the atmosphere – makes it possible to improve cementation while shortening the process. Here, the variation of temperature plays an important role if the problem is

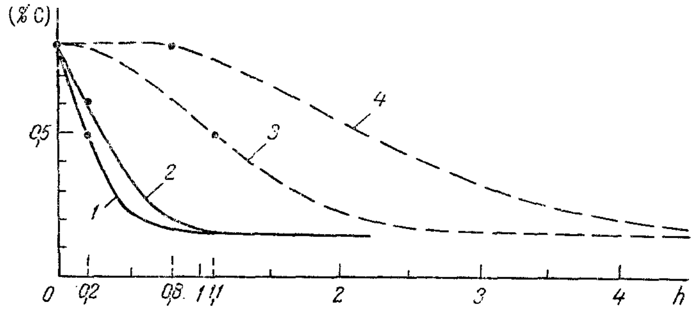

Fig. 3. Optimum (with respect to time) cementation profiles of two types with "smooth" control: 1) type A, for set of control parameters No. 1 (Table 3); 2) type A, for set No. 2; 3) type B, for set No. 3; 4) type B, for set No. 4.

TABLE 3. Optimum (with respect to time) Values of Control Parameters with "Smooth" Control

t_2, °C	τ^{tot}, h	$\hat{q}$, $\frac{C_{sur}}{h_{gr}/C_{gr}}$	C^{atm}_{max}, % C	C^{atm}_{min}, % C	t_1, °C	τ_1, h	τ_2, h	No. of set
840	1,2	$\frac{0,8}{0,2/0,5}$	1,12	0,89	938,1	0,5	1,0	1
	1,2	$\frac{0,8}{0,2/0,6}$	1,16	0,85	1041,3	0,3	1,2	2
	11,0	$\frac{0,8}{1,1/0,5}$	1,14	0,80	964,3	5,6	10,3	3
	20,0	$\frac{0,8}{0,8/0,8}$	1,16	0,80	1038,0	12,0	12,3	4

solved within a sufficiently broad range of $\hat{q}$. The simplest control variant, proposed earlier in [1], yields a satisfactory result only with small values of h_{gr}, C_{gr}: $h_{gr} \leq 0.2$ mm, $C_{gr} \leq 0.6\%$. In the other cases, the cementation time which is optimum with respect to the duration of the process is clearly inadequate for obtaining the required effect.

The optimum values of the parameters of the model [1] which correspond to this duration lead to cementation profiles characterized by a distinct maximum near the surface of the specimen. The value of this maximum reaches 20% of the prescribed value of concentration on the surface. A halving of this maximum (with other τ^{tot}_{min}) leads to deviation of the concentration profile from the subsurface reference point by as much as 50% C_{gr}.

5. A problem involving optimization of saturation time without loss of quality can be formulated similarly to (2) with the inclusion of the duration of the process τ^{tot} as one of the control parameters: $\hat{p}_{min} = (\hat{p}, \tau^{tot})$. However, as was shown by our numerical experiments, automatic selection of $\hat{p}_{min}$ leads to a substantial increase in processor operating time in the computer. A similar effect is produced by refinement of the grid $\{\tau^{tot}_{\ell}\}$ in the algorithm proposed above for the solution of problem (4).

Thus, it turns out to be useful to construct nomograms that compare the shortest optimum times τ^{tot}_{min} for different sets of reference points $\hat{q}$. Then the search for the optimum (with respect to time) regime for each $\hat{q}$ reduces to the solution of problem (2) with a fixed cementation time which is known beforehand to be optimum.

Since the equation whose solution is the sought relation [7] is given in implicit form, to construct such nomograms we use criteria (a), (b), and (c) to evaluate the set of cementation profiles obtained from the solution of problem (2) on the grid $\{\tau^{tot}_{\ell}\}$ for each $\hat{q}$. This allows us to obtain a clear representation of the relation $\tau^{tot}_{min} = \tau^{tot}_{min}(\hat{q})$.

The nomograms we constructed were based on the family of curves $h_{gr} = h_{gr}(\tau^{tot}_{min})$, each of which corresponds to a value of C_{gr}.

In the semi-logarithmic scale $\tau^{tot}_{min} \to \log \tau^{tot}_{min}$, these curves are represented by line segments (see Fig. 4b). Figure 4a shows the relations $h_{tot} = h_{tot}(h_{gr})$. As was noted above, the point (h_{tot}, $C_{gr} = \hat{C}_0 + 0.05\%$) is not among the reference points, while the values of

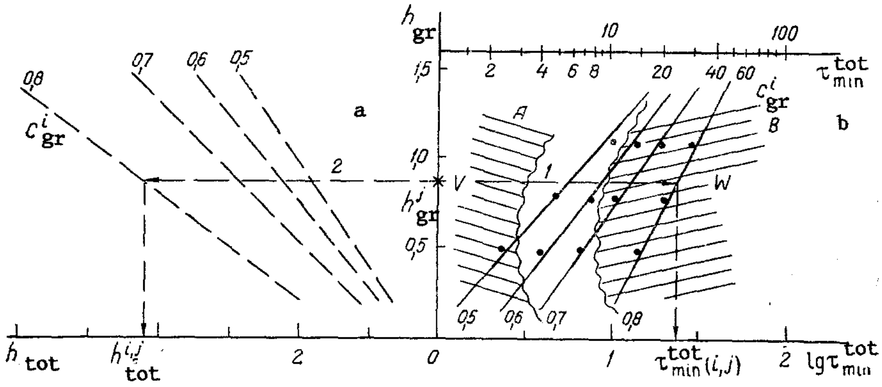

Fig. 4. Nomograms of the relation $h_{gr}-\tau^{tot}_{min}-h_{tot}$ for several values of concentration C_{gr} (numbers next to the curves) and the scheme in which they are used. h_{tot}, h_{gr}, mm; τ^{tot}_{min}, h.

h_{tot} are obtained by solving problem (2). Thus, Fig. 4a provides additional information on the total saturation depth that can be expected in the optimum regime.

Finally, the use of the above criteria makes it possible to determine on a sheet of nomograms (Fig. 4) the regions (τ^{tot}_{min}, $\hat{q}$) in which one can expect profiles of types A and B, respectively. (Each such sheet corresponds to one value of C_{sur}: C_{sur} = 0.8%).

Figure 4 also shows a possible scheme for the use of the nomogram with one of the required parameter values.

Suppose that we need to know the optimum cementation time in order to obtain the concentration C^i_{gr} at the edge of a layer of the thickness h^j_{gr}. Let the search be represented in this scheme by the dashed line 1 originating at point V on the axis h_{gr}. Movement along the horizontal line to the line corresponding to the prescribed C^i_{gr} and subsequent "descent" on the axis τ^{tot}_{min} allows us to find the sought value $\tau^{tot}_{min}(i, j)$. Here, we could also find $h^{i,j}_{tot}$ by moving from point V to the left on dashed line 2 until we reach the prescribed numerical value of C^i_{gr}. We simultaneously establish three concentration profiles. In the example being discussed, point W in the scheme belongs to a region of type B, and we obtain a "plane" near-surface section of thickness $\le h^j_{gr}$.

The value found, $\tau^{tot}_{min}(i, j)$, can be used by the operator on the computer to calculate the optimum value of $\hat{p}_{min}$ corresponding to the required $\hat{q} = \{C_{sur}; h^j_{gr}, C^i_{gr}\}$ with the aid of the algorithm that solves problem (2).

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